Outgrowths of the DLMF Project: **Part 2**: NIST Digital Repository of Mathematical Formulae

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Challenges in 21st Century: Experimental Mathematical Computation

Institute for Computational and Experimental Research in Mathematics, Providence, Rhode Island

July 21, 2014

Digital Repository of Mathematical Formulae

- **Online compendium** of **mathematical formulae**
  - **orthogonal polynomial** and **special function formulae**
- **DRMF** attempts to use **Web 2.0 technologies** to move beyond the **static presentation** of **reference data** to a **platform** that **encourages community interaction** and **collaboration**.
- **DRMF** utilizes of **DLMF LaTeX macros**
  - tie **specific character sequences** to **well-defined mathematical objects**.
  - Provides an **internet link** to **standard**, **precise** orthogonal polynomial and special function **definitions** through the DLMF and DRMF
- Uses **MediaWiki** wiki software
  - **MathML** support
  - **LaTeXML**
  - **MathJax**

DRMF goals

The **DRMF** will be designed for a **mathematically literate audience** and should:

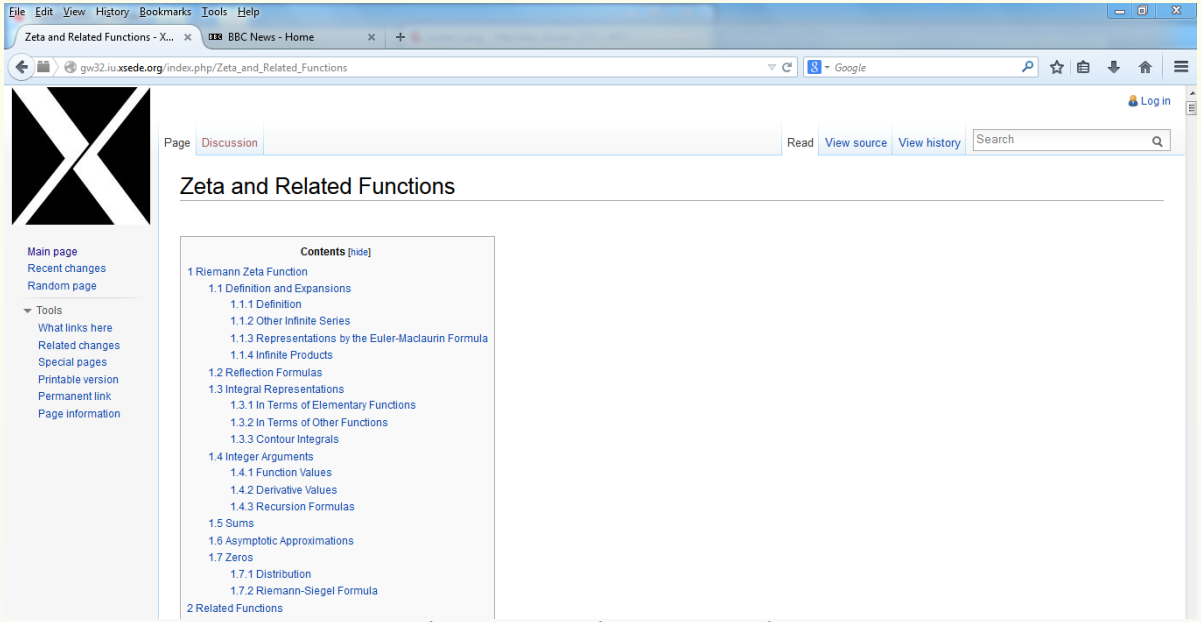
1. **facilitate interaction** among a **community of mathematicians and scientists** interested in **formulae data** related to **orthogonal polynomials and special functions** (OPSF);
2. be **expandable,** allowing the input of new formulae;
3. be **accessible as a standalone resource**;
4. have a **user friendly, consistent, and hyperlinkable viewpoint and authoring perspective**; and
5. contain **easily searchable mathematics** and take advantage of modern **MathML tools** for **easy to read**, **scalably rendered mathematics**.

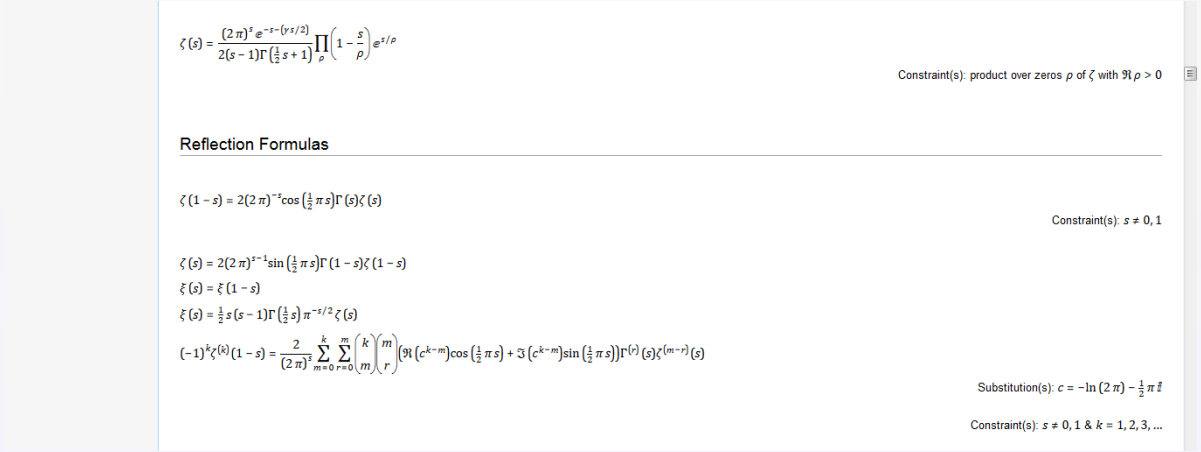
DRMF Seeding Projects

Math OCR, Macro replacement, and Wikitext generation, to implement pre-existing book compendia

- **Mathematical Optical Character Recognition** project
  - **Bateman manuscript project**: Higher Transcendental Functions, Tables of Integral Transforms
  - **Byrd & Friedman**'s Handbook of Elliptic Integrals for Engineers and Scientists
- **DLMF LaTeX macro replacement** project
  - Hypergeometric Orthogonal Polynomials and Their q-Analogues – **KLS**
  - **KLS addendum** by Tom Koornwinder
  - future?: **Andrews, Askey & Roy** : Special Functions
  - future?: **Ismail** : Classical and Quantum Orthogonal Polynomials in One Variable
  - future?: etc.
- **Wikitext generation** project
  - NIST Digital Library of Mathematical Functions (ch. 25) : 170 formulas

DRMF Zeta and Related Functions Page ... DLMF Wikitext





DLMF macros provide semantic content in formulas

- **DLMF OPSF Macros** via **LaTeXML-server**
  - 546 **semantic DLMF** LaTeX **OPSF** macros
  - **additional** 38 **semantic LaTeX macros**
- **Objects:** `\sum,\int,\deriv{f}{x},\qderiv[n]{q}@{z}`
- **Constants:** `\expe,\iunit,\cpi,\EulerConstant`
- **Special Functions and Orthogonal Polynomials**

$\Gamma(z)$	`\EulerGamma@{z}`	`http://dlmf.nist.gov/5.30#E1`
$J_\nu(z)$	`\BesselJ{\nu}@{z}`	`http://dlmf.nist.gov/10.2#E2`
$Q_\nu^\mu(z)$	`\LegendreQ[\mu]{\nu}@{z}`:	`http://dlmf.nist.gov/14.3#E7`
$P_n^{(\alpha,\beta)}(x)$	`\JacobiP{\alpha}{\beta}{n}@{x}`	`http://dlmf.nist.gov/18.3#T1.t1.r3`

Mathematical Formulae

- Whereas **Wikipedia** and other web authoring tools **manifest notions or descriptions** as **first class objects**, the DRMF does that with **mathematical formulae**.
- **DRMF** provides for each formula, a **formula home page**:
  1. **Rendered description of the formula** (required);
  2. **Constraints** the formula must obey
  3. **Substitutions** required to understand formula;
  4. **Bibliographic citation** (required);
  5. Open section for **proofs** (required) – *DLMF*;
  6. **List of symbols** and **links** to definitions (required) – *DLMF macros*;
  7. Open section for **notes** – *connections between formulas*; and
  8. Open section for **external links** – *computer generated proofs*;

Sample formula home page

HowardCohl Talk Preferences Watchlist Contributions Log out

Page Discussion Read Edit View history Search

Formula:DLMF:25.4:E5

<< Formula:DLMF:25.4:E4 formula in Zeta and Related Functions Formula:DLMF:25.5:E1 >>

$$(-1)^k\zeta^{(k)}(1-s)=\frac{2}{(2\pi)^s}\sum_{m=0}^{k}\sum_{r=0}^{m}\binom{k}{m}\binom{m}{r}\left(\Re\left(e^{k-m}\right)\cos\left(\tfrac{1}{2}\pi s\right)+\Im\left(e^{k-m}\right)\sin\left(\tfrac{1}{2}\pi s\right)\right)\Gamma^{(r)}(s)\zeta^{(m-r)}(s)$$

Contents [hide]

Substitution(s) [edit]

$c=-\ln(2\pi)-\frac{1}{2}\pi\,\mathbf{i}$

Constraint(s) [edit]

$s\neq 0,1\ \&\ k=1,2,3,\ldots$

Proof [edit]

We ask users to provide proof(s), reference(s) to proof(s), or further clarification on the proof(s) in this space.

Symbols List [edit]

(-1) : (-1) = \opminus : http://dlmf.nist.gov/5.7.E7

ζ : Riemann zeta function : http://dlmf.nist.gov/25.2#E1

π : the ratio of a circle's circumference to its diameter : http://dlmf.nist.gov/5.19.E4

$\binom{a}{b}$: binomial coefficient : http://dlmf.nist.gov/1.2#E1 http://dlmf.nist.gov/26.3#SS1.p1

$\Re\ a$: real part : http://dlmf.nist.gov/1.9#E2

$\cos$: cosine function : http://dlmf.nist.gov/4.14#E2

$\Im\ a$: imaginary part : http://dlmf.nist.gov/1.9#E2

$\sin$: sine function : http://dlmf.nist.gov/4.14#E1

Γ : Euler's gamma function : http://dlmf.nist.gov/5.2#E1

$\ln$: principal branch of logarithm function : http://dlmf.nist.gov/4.2#E2

$\mathbf{i}$: imaginary unit : http://dlmf.nist.gov/1.9.i

Bibliography [edit]

Equation (5), Section 25.4 of **DLMF**.

URL links [edit]

We ask users to provide relevant URL links in this space.

<< Formula:DLMF:25.4:E4 formula in Zeta and Related Functions Formula:DLMF:25.5:E1 >>

Further questions

- How does one **facilitate** effective **community interaction** & **contribution** with such a **resource**?
  - implement a high degree of **computer verification** of community input
  - ensure a degree of **moderation** in the wiki
- Can one build a piece of **intelligent software** which is able to
  - scan in **books**;
  - produces **LaTeX source**;
  - replaces commands for functions in the source with **semantic macros**;
  - **extracts data** from the **text** (such as constraints)
  - **associates data** with relevant formulae and **removes** text;
  - **produces Wikitext**;
  - and **uploads** Wikitext to a publicly accessible website?
- How does one **search** the resulting mathematical database?

Ongoing projects to investigate the above questions

- **Macro replacements** from well-constructed LaTeX source
- **Extraction** of **mathematical data** from **text** (keywords)
- **Wikitext generation**
- Porting the **DLMF search engine** in **MediaWiki** (DRMF)
- **Output of formula data** from **right-clickable menus** in a variety of **formats** so that formulas can be **used** and also **verified**
  - LaTeX expanded
  - LaTeX semantic
  - presentation MathML
  - content MathML
  - Mathematica
  - Maple
  - Sage

Virtual machines

Virtual Machine Instances:

- **XSEDE** project
  - 2 **XSEDE** CentOS: **demo and deployment**
  - 2 **XSEDE** Ubuntu **server**: **LaTeXML, Mathoid**
- **Wikimedia Foundation – Wikitech**
  - 4 **WMF Vagrant** instances

Acknowledgements

- **Moritz Schubotz** (TU-Berlin): **MediaWiki Math**
- **Bruce Miller** (NIST) : **DLMF Macros**
- **Janelle Williams** (VSU) : **2013 SURF student**
- **High School Students:**
  - **Jake Migdall** – MathJax menu customization
  - **Cherry Zou** – seeding/macro replacement
  - **Alex Danoff** – seeding/macro replacement
  - **Amber Liu** – MathJax menu customization
  - **Jimmy Li** – mathematical search

Poster session with website demos : Wed. evening

NIST Digital Repository of Mathematical Formulae (DRMF)

Howard S. Cohl*, Marjorie A. McClain*, Bonita V. Saunders*,
Moritz Schubotz§, Janelle C. Williams†

*NIST/ACMD, §Technische Universität Berlin (Germany), †Virginia State University

- A **digital compendium of math formulae** for **orthogonal polynomials and special functions** and **associated math data**. Uses **Web 2.0** technologies to move beyond the **static presentation of reference data** to a **platform** that encourages **community interaction** and **collaboration.**
- **Offshoot** project of **NIST Digital Library of Math Functions (DLMF)** using **MediaWiki** and **MediaWiki Math extension**. **Math rendering** in **MediaWiki** using **MathML. Menuing** support using **MathJax**. **MediaWiki extension development** using **PHP, JavaScript** & **Java**.
- Use of **DLMF semantic LaTeX macro set** for **special functions and orthogonal polynomials** e.g.,

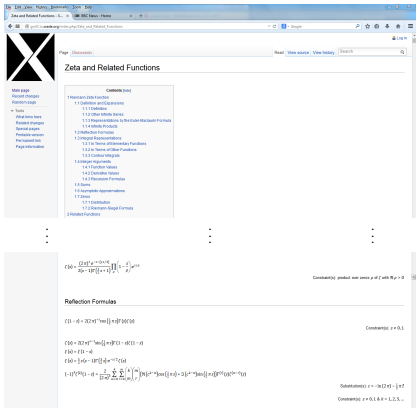
$\Gamma(z)$	`\EulerGamma@{z}`	`http://dlmf.nist.gov/5.2#E1`
$P_n^{(\alpha,\beta)}(x)$	`\JacobiP{\alpha}{\beta}{n}@{x}`	`http://dlmf.nist.gov/18.3#T1.t1.r3`

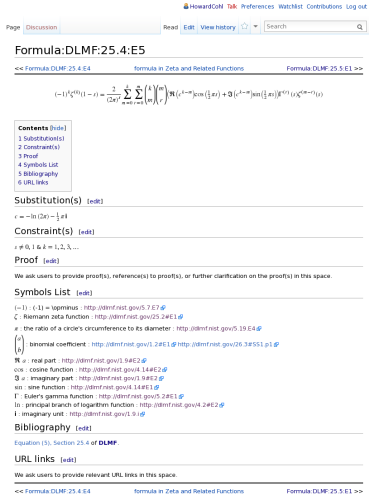
- **DRMF** treats **formulae** as **first class objects**, describing them in **formula home pages** which contain:

1. **Rendered description** of the **formula** (required);
2. **Bibliographic citation** (required);
3. Open section for **proofs** (required);
4. **List of symbols** and **links** to definitions (required);
5. Open section for **notes**;
6. Open section for **external links**;
7. **Substitutions** required to understand formula; and
8. **Constraints** the formula must obey.

- **Wikitext** generation and **semantic DLMF LaTeX macro replacement** effort using **IDL** & **Python**.
  1. **DLMF LaTeX macros** already implemented for the **DLMF Zeta** chapter;
  2. Hypergeometric **Orthogonal Polynomials** and their q-Analogues + **KLS addendum**.

Zeta and Related Functions Page





Formula Home Page ⟶

`http://gw32.iu.xsede.org/index.php/Main_Page`

13th International Symposium on Orthogonal Polynomials, Special Functions & Applications

June 1-5, 2015
National Institute of Standards and Technology
Gaithersburg, Maryland, USA

`http://www.siam.org/meetings/opsfa13`

`http://www.nist.gov/itl/math` `https://wis.kuleuven.be/events/OPSFA/Steering`

Plenary Speakers:
- Percy Deift, Courant Institute of Mathematical Sciences, New York University, USA
- Charles F. Dunkl, University of Virginia, USA
- Olga Holtz, Technische Universität Berlin, Germany
- Mourad E.H. Ismail, University of Central Florida, USA
- Teresa E. Pérez Fernández, Universidad de Granada, Spain
- Sarah Post, University of Hawaii at Manoa, USA
- Nico Temme, Centrum Wiskunde & Informatica (CWI), The Netherlands
- Craig A. Tracy, University of California Davis, USA
- Lauren Williams, University of California Berkeley, USA
- Wadim Zudilin, The University of Newcastle, Australia
- Alexei Zhedanov, Donetsk Institute for Physics and Technology, Ukraine

Themes: **Orthogonal Polynomials and Special Functions**, including aspects within:
- classical analysis
- approximation theory
- continued fractions
- potential theory
- q-calculus
- asymptotics
- Riemann-Hilbert problems
- random matrix theory
- superintegrability and supersymmetry
- integrable systems
- Painlevé equations
- orthogonal polynomials and special functions of several variables
- orthogonal polynomials associated with root systems
- spherical functions
- orthogonality on the complex plane
- multiple orthogonal polynomials
- Sobolev orthogonal polynomials
- stochastic processes

and connections to other disciplines, including:
- science and industry
- handbooks
- numerical algorithms and tables
- symbolic computation
- combinatorics
- number theory
- theoretical physics
- probability theory and statistics

Organizing Committee
- Diego Dominici, State University of New York at New Paltz, USA
- Daniel W. Lozier, National Institute of Standards and Technology, USA

Scientific Committee:
- Richard A. Askey, University of Wisconsin, USA
- Howard S. Cohl, National Institute of Standards and Technology, USA
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- Victor H. Moll, Tulane University, USA
- Adri Olde Daalhuis, The University of Edinburgh, UK
- Audrey Terras, University of California San Diego, USA
- Walter Van Assche, Katholieke Universiteit Leuven, Belgium
- Luc Vinet, University of Montreal, Canada

Outgrowths of the Digital Library of Mathematical Functions Project Part 1

DLMF Standard Reference Tables

http://dlmftables.uantwerpen.be

Daniel Lozier

NIST, Gaithersburg, MD, USA

Outline

- Introduction
  - DLMF
  - DLMF Standard Reference Tables
  - Algorithms and software
- Examples
  - Table generation
  - Table comparison
- Acknowledgements

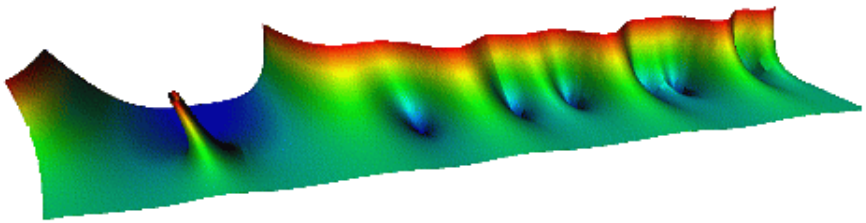
http://dlmf.nist.gov

Digital Library of Mathematical Functions

- Index
- Notations
- Search
- Need Help?
- How to Cite
- Customize

About the Project

NIST National Institute of Standards and Technology



NIST Digital Library of Mathematical Functions

Project News

2012-03-23 *DLMF Update; Version 1.0.4*
2011-10-19 *Digital Library of Mathematical Functions Team Cited for IT Innovation*

- *More news*

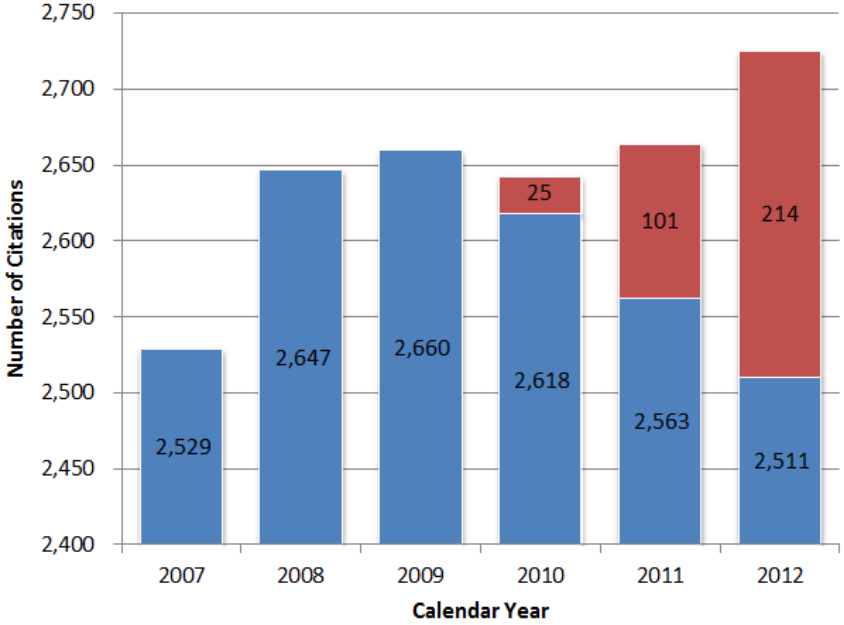
NIST Digital Library of Math Fcns

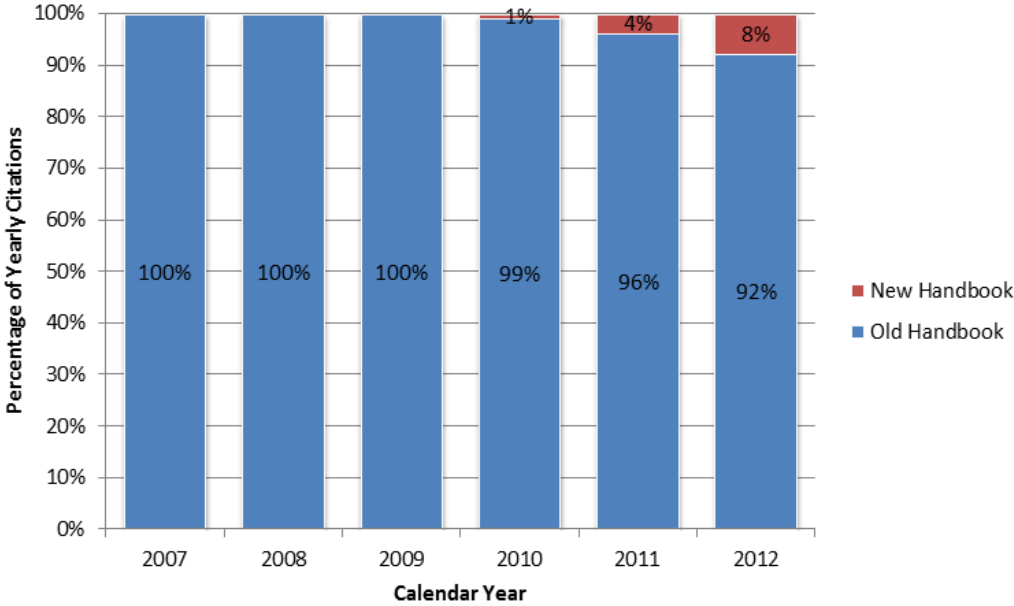
and

NIST Handbook of Math Fcns

compared to

Abramowitz and Stegun, 1964





cantlive.ua.ac.be/dlmftables/

- Help
- Background
- Disclaimer

DLMF Tables

Welcome to ***the Prototype of*** DLMF Standard Reference Tables!

The DLMF Standard Reference Tables service aims to provide tables of numerical values for special functions, with guaranteed accuracy to high precision and computed over parameters and arguments of your choice.

DLMF Std. Reference Tables *Accurate function values on demand*

- Keyed to http://dlmf.nist.gov
- Web input of
  - Desired arguments
  - Desired precision
- Web output of
  - Enclosures computed to 5 digits higher precision or one of 5 rounding modes at desired precision

DLMF Std. Reference Tables *Accuracy tests of function values from other sources*

- Web input of
  - Arguments and function values from an external source
- Web output of
  - Digit-by-digit comparison
  - Erroneous digits shown in red
  - Relative error

Algorithms and Software

- Special functions computed to arbitrary precision and guaranteed accuracy
- Real variables, decimal or binary arithmetic
- Algorithms depend on detailed error analysis with series and continued fractions
- For details see
  - A. Cuyt et al, *Handbook of Continued Fractions*, Springer, 2008
  - F. Backeljauw et al, *Validated Evaluation of Special Mathematical Functions*, Science of Computer Programming, v. 90A, 2014, pp. 2-20

Example

table generation

Bessel function $J_n(x)$,
integer order, real argument

Table Request

Choose a function Bessel function J

$J_\nu(x)$ denotes the Bessel function of the first kind of order ν; see DLMF (10.2.2). Computations are currently restricted to integer and half-integer orders ν, and real values of x.

Evaluate $J_\nu(x)$ as a ***Tabulation*** or ***Comparison***?

Get function arguments from ***form*** or ***data file***?

Specify grid for function parameters and arguments:

Choose ν from list

0 5 10 15 20 30 40 50 100

Choose x from list

1

Show output to 25 digits, using

interval mode

Computed values of $J_\nu(x)$

Computed using MpIeee, for ν being each of 0, 5, 10, 15, 20, 30, 40, 50, 100 and x being 1.0; output to 25 digits, using interval mode; computation and output in base 10 .

ν	x	$J_\nu(x)$
0	1.0	7.65197 68655 79665 51449 7175 26103±08 $\times 10^{-1}$
5	1.0	2.49757 73021 12344 31375 0655 40988±03 $\times 10^{-4}$
10	1.0	2.63061 51236 87453 20699 7853 68779±03 $\times 10^{-10}$
15	1.0	2.29753 15322 10344 44380 6556 20688±03 $\times 10^{-17}$
20	1.0	3.87350 30085 24657 71891 4787 52995±04 $\times 10^{-25}$
30	1.0	3.48286 97942 51482 90224 9676 48686±04 $\times 10^{-42}$
40	1.0	1.10791 58511 28632 66217 5020 74447±02 $\times 10^{-60}$
50	1.0	2.90600 49481 73239 39446 9381 57792±03 $\times 10^{-80}$
100	1.0	8.43182 87896 26708 54923 5063 65845±09 $\times 10^{-189}$

Example

table comparison

Matlab's error function $\mathrm{erf}(x)$,
real argument

Table Request

Choose a function error function

$\operatorname{erf} x$ denotes the error function; see DLMF (7.2.1).
Computations are currently restricted to real values of x.

Evaluate $\operatorname{erf} x$ as a ***Tabulation*** or ***Comparison***?

Specify the data file to compare to:
Browse... Matlab-erf-6args-6vals-input-file.txt

Note: The lines in the data file must contain values for x and $\operatorname{erf} x$ in that order, separated by commas or spaces.

Show output to the number of digits in file, using
round to nearest (even)

Computed values of erf x

Computed using MpIeee(on odd lines), compared to x, erf x (on even lines), for x, erf x from file Matlab-erf-6args-6vals-input-file.txt; output to 17 digits, using round to nearest (even); computation and output in base 10 .

x	erf x	Relative Error
$6.25000\,00000\,00000\,0\times10^{-2}$	$7.04319\,77722\,38707\,8\times10^{-2}$	
	$7.04319\,77722\,38707\,4\times10^{-2}$	$5.7\ \times10^{-17}$
$1.25000\,00000\,00000\,0\times10^{-1}$	$1.40316\,20480\,13338\,2\times10^{-1}$	
	$1.40316\,20480\,13338\,0\times10^{-1}$	$1.5\ \times10^{-16}$
$1.87500\,00000\,00000\,0\times10^{-1}$	$2.09117\,67705\,93758\,5\times10^{-1}$	
	$2.09117\,67705\,93758\,4\times10^{-1}$	$4.8\ \times10^{-17}$
$2.50000\,00000\,00000\,0\times10^{-1}$	$2.76326\,39016\,82369\,3\times10^{-1}$	
	$2.76326\,39016\,82369\,6\times10^{-1}$	$1.1\ \times10^{-16}$
$3.12500\,00000\,00000\,0\times10^{-1}$	$3.41468\,63350\,15950\,1\times10^{-1}$	
	$3.41468\,63350\,15950\,1\times10^{-1}$	0.0
$3.75000\,00000\,00000\,0\times10^{-1}$	$4.04116\,90943\,48223\,0\times10^{-1}$	
	$4.04116\,90943\,48223\,1\times10^{-1}$	$2.5\ \times10^{-17}$

Acknowledgements

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  - *Bruce Miller*
  - *Marje McClain*
  - *Bonita Saunders*
- Primary Antwerp Collaborators
  - *Franky Backeljauw*
  - *Stefan Becuwe*
  - *Annie Cuyt*

Feedback

help us provide an excellent data service!!

- Digit display
  - *Do you like or dislike?*
  - *Do you prefer just relative errors?*
- Extra digits and color
  - *Do you like or dislike?*
- Please send ideas for
  - *Further development*
  - *Further application areas*